Homework

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March 24, 2021

1 作業

1. 求下列$\mathcal{R}^2$中子集的內部、邊界與閉包。

   (a) $S = \{(x, y)|x > 0, y \neq 0\}$

   (b) $S = \left\{(x, y)|0 < x^2 + y^2 \leq 1\right\}$

   (c) $S = \left\{(x, y)|0 < x \leq 1, y = \sin\frac{1}{x}\right\}$

2. 求下列點集的全部聚點。

   (a) $S = \left\{(-1)^k\frac{k}{1+k}|k = 1, 2, \cdots,\right\}$

   (b) $S = \left\{(-1)^k\frac{k}{1+k}|k = 1, 2, \cdots,\right\}$

   (c) $S = \left\{\left(\cos\frac{2k\pi}{5}, \sin\frac{2k\pi}{5}\right)|k = 1, 2, \cdots,\right\}$

   (d) $S = \left\{(x, y)|(x^2 + y^2)(y^2 - x^2 + 1) \leq 0\right\}$

3. 證明康托閉區域套定理：設$\{S_k\}$是$\mathcal{R}^n$上的非空閉集序列，滿足：

   $$S_1 \supset S_2 \supset \cdots \supset S_k \supset S_{k+1} \supset \cdots$$

   以及$\lim \mathrm{diag} S_k = 0$，則存在唯一一點屬於$\bigcap\limits_{k=1}^{\infty} S_k$. 這裡

   $$\mathrm{diag} S = \sup\{|x - y||x, y \in S\},$$

   稱為S的直徑。

4. 求下列函數的極限

(a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^2+y^2}$

(b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{1+x^2+y^2}{x^2+y^2}$

(c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{1+x^2+y^2}-1}$

(d) $\lim\limits_{(x,y)\to(0,0)} (x+y)\sin\dfrac{1}{x^2+y^2}$

(e) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$

5. 討論下列函數在點$(0,0)$的重極限與累次極限

(a) $f(x,y)=\dfrac{y^2}{x^2+y^2}$

(b) $f(x,y)=(x+y)\sin\dfrac{1}{x}\sin\dfrac{1}{y}$

(c) $f(x,y)=\dfrac{x^2y^2}{x^2y^2+(x-y)^2}$

(d) $f(x,y)=y\sin\dfrac{1}{x}+x\sin\dfrac{1}{y}$

6. 討論下列函數的連續性

(a) $f(x,y)=\tan\left(x^2+y^2\right)$

(b) $f(x,y)=\lfloor x+y\rfloor$

(c) $f(x,y)=\begin{cases} \frac{\sin xy}{\sqrt{x^2+y^2}}, & x^2+y^2\neq 0 \\ 0, & x^2+y^2=0 \end{cases}$

(d) $f(x,y)=\begin{cases} 0, x \text{ is an irrational number} \\ y, x \text{ is a rational number} \end{cases}$