

測試三—導數

武國寧

1 求下列函數的導數

1. $y = e^{2x} \sin 3x$

2. $y = \sqrt{1+x} - \ln(x + \sqrt{x+1})$

3. $y = \arcsin(e^{-x^2})$

4. $y = x\sqrt{a^2 - x^2} + \frac{x}{a^2 - x^2}$

5. $y = \ln \sin x$

6. $y = \ln(x + \sqrt{x^2 + a^2})$

7. $y = \frac{1}{2} \left(x\sqrt{a^2 - x^2} + a^2 \arcsin \frac{x}{a} \right)$

8. $y = x^x$

9. $y = x^{x^a} + x^{a^x} + a^{x^x}$

10. $y = |x^3(x-1)(x+7)|$

2 設 $f(x)$ 可導，求下列函數的導數

1. $f\left(\frac{1}{\ln x}\right)$

2. $\arctan f(x)$

3. $f(f(e^{x^2}))$

4. $f\left(\frac{1}{f(x)}\right)$

5. $\frac{1}{f(f(x))}$

3 求下列隱函數的導數

1. $y = x + \arctan y$

2. $y + xe^y = 1$

3. $x^3 + y^3 - 3axy = 0$

4. $2y \sin x + x \ln y = 0$

4 求下列參數形式函數的導數

1. $\begin{cases} x = at^2 \\ y = bt^3 \end{cases}$

2. $\begin{cases} x = \sinh at \\ y = \cosh bt \end{cases}$

5 求下列函數的 n 階導數 $y^{(n)}$

1. $y = \sin^2 \omega x$

2. $y = x^2 e^x$

3. $y = \frac{1}{x^2 - 5x + 6}$

4. $y = e^{ax} \cos \beta x$

6 求下列參數形式函數的二階導數 $\frac{d^2y}{dx^2}$

1. $\begin{cases} x = at^2 \\ y = bt^3 \end{cases}$

2. $\begin{cases} x = t(1 - \sin t) \\ y = t \cos t \end{cases}$

7 利用反函數的求導數公式 $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$ 證明:

1. $\frac{d^2x}{dy^2} = -\frac{y''}{(y')^3}$

2. $\frac{d^3x}{dy^3} = \frac{3(y'')^2 - y'y'''}{(y')^5}$