《數學分析I》不定積分，定積分測試題

武國寧

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班級_____ 姓名_____ 學號_____

求下列不定積分

1. $\displaystyle\int x \arcsin x \,\mathrm{d}x$

2. $\displaystyle\int e^{\sin x} \sin 2x \,\mathrm{d}x$

3. $\displaystyle\int e^{\sqrt{x}} \,\mathrm{d}x$

4. $\displaystyle\int \frac{1 - \tan x}{1 + \tan x} \,\mathrm{d}x$

5. $\displaystyle\int \frac{1}{\cos^4 x} \,\mathrm{d}x$

6. $\displaystyle\int \frac{\sin x}{\sin x + \cos x} \,\mathrm{d}x$

7. $\displaystyle\int \frac{\cos x}{\sin x + \cos x} \,\mathrm{d}x$

8. $\displaystyle\int \frac{1}{1 + x^4} \,\mathrm{d}x$

9. $\displaystyle\int \frac{1 + x^2}{1 + x^4} \,\mathrm{d}x$

10. $\displaystyle\int \frac{1}{x(1+x)(1+x^2)}\,\mathrm{d}x$

11. $\displaystyle\int \frac{\arcsin x}{x^2}\,\mathrm{d}x$

12. $\displaystyle\int \sin^n x\,\mathrm{d}x$

13. $\displaystyle\int \tan^n x\,\mathrm{d}x$

利用定積分的定義求極限

1. $\displaystyle\lim_{n\to+\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right)$

2. $\displaystyle\lim_{n\to+\infty} n\left(\frac{1}{n^2+1^2}+\frac{1}{n^2+2^2}+\cdots+\frac{1}{n^2+n^2}\right)$

計算下列定積分

1. $\displaystyle\int_0^1 \frac{1-x^2}{1+x^2}\,\mathrm{d}x$

2. $\displaystyle\int_e^{e^2} \frac{1}{x\ln x}\,\mathrm{d}x$

3. $\displaystyle\int_4^9 \sqrt{x}+\frac{1}{\sqrt{x}}\,\mathrm{d}x$

4. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^5 x\sin 2x\,\mathrm{d}x$

5. $\displaystyle\int_0^1 \sqrt{4-x^2}\,\mathrm{d}x$

6. $\displaystyle\int_0^1 \frac{1}{e^x+e^{-x}}\,\mathrm{d}x$

7. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}\,\mathrm{d}x$

8. $\displaystyle\int_{\frac{1}{e}}^{e} \ln|x|\,\mathrm{d}x$

9. $\displaystyle\int_{0}^{\frac{\pi}{2}} |\sin x - \cos x|\,\mathrm{d}x$

10. $\displaystyle\int_{0}^{5} \lfloor x \rfloor\,\mathrm{d}x$

11. $\displaystyle\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x}\,\mathrm{d}x$

12. $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin^n x\,\mathrm{d}x$

13. $\displaystyle\int_{0}^{1} x^m(1-x)^n\,\mathrm{d}x$

計算下列極限

1. $\displaystyle\lim_{x\to 0} \frac{1}{x}\int_{0}^{x} \cos t^2\,\mathrm{d}t$

2. $\displaystyle\lim_{x\to\infty} \frac{\left(\int_{0}^{x} e^{t^2}\right)^2}{\int_{0}^{x} e^{2t^2}\,\mathrm{d}t}$

證明題

1. $\displaystyle\int_{0}^{\frac{\pi}{2}} f(\sin x)\,\mathrm{d}x = \int_{0}^{\frac{\pi}{2}} f(\cos x)\,\mathrm{d}x$

2. $\displaystyle\int_{0}^{\pi} x f(\sin x)\,\mathrm{d}x = \frac{\pi}{2}\int_{0}^{\pi} f(\sin x)\,\mathrm{d}x$